Automated decentralized modal analysis using smart sensors

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SUMMARY

Understanding the dynamic behavior of civil engineering structures is important to adequately resolve problems related to structural vibration. The dynamic properties of a structure are commonly obtained by conducting a modal survey that can be used for model updating, design verification, and improvement of serviceability. However, particularly for large-scale civil structures, modal surveys using traditional wired sensor systems can be quite challenging to carry out due to difficulties in cabling, high equipment cost, and long setup time. Wireless smart sensor networks (WSSN) offer a unique opportunity to overcome such difficulties. Recent advances in sensor technology have realized low-cost smart sensors with on-board computation and wireless communication capabilities, making deployment of a dense array of sensors on large civil structures both feasible and economical. However, as opposed to wired sensor networks in which centralized data acquisition and processing are a common practice, the WSSN requires decentralized algorithms due to the limitation associated with wireless communication; to date such algorithms are limited. This paper proposes a new decentralized hierarchical approach for modal analysis that reliably determines the global modal properties and can be implemented on a network of smart sensors. The efficacy of the proposed approach is demonstrated through several numerical examples. Copyright © 2009 John Wiley & Sons, Ltd.

KEY WORDS: decentralized modal analysis; smart sensor; wireless smart sensor network; sensor topology; automated sensor network

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1. INTRODUCTION

Research on structural vibration issues started in 1930s focusing on understanding the
dynamic behavior of aircraft [1]; since that time, principles in structural dynamics have been
widely considered in other applications. Identified dynamic characteristics are often used for
model updating, design verification, and serviceability improvement. A typical example is the
retrofit of the London Millennium Footbridge to address unexpected large lateral
vibrations caused by pedestrians [2]. To control bridge vibrations, fluid-viscous dampers and
tuned mass dampers were installed on the bridge. In designing this damping system,
understanding the dynamic characteristics of the as-built bridge was critical to ensuring
efficient operation. The dynamic properties of structures are commonly obtained by
conducting a modal survey that typically consists of measuring vibration signals of the
structure and analyzing the measured data to estimate modal parameters such as natural
frequencies, mode shapes, modal damping factors, and modal masses. However, for large-
scale civil engineering structures, the long setup time, difficulties in cabling, and high
equipment cost [3] make conducting a modal survey using the conventional wired sensors
challenging. For instance, imagine a modal survey of the Golden Gate Bridge. Miles
of cables would be required to connect the central base station to the sensor nodes
distributed along the towers and deck; installation would be both costly and time-consuming.
Thus, more efficient means for modal surveys are desired particularly for large-scale civil
structures.

Wireless smart sensor networks (WSSN) provide a promising alternative to traditional
wired sensor networks. Recent advances in sensor technology have enabled low-cost smart
sensors that have wireless communication, on-board processing, and multi-scale sensing
capabilities; even lower prices are expected once smart sensors are in mass production. A variety
of commercially available smart sensors have been developed in the last decade [4], opening a
new era in structural health monitoring (SHM). Although some challenges, such as power
consumption and long-term reliability, still need to be resolved for the smart sensors to be more
widely adopted, available smart sensors are already quite capable and can be expected to
replace the traditional wired sensors for many applications. One particularly critical challenge
relates to how the sensor data are processed. Centralized data acquisition and processing
schemes (See Figure 1(a)) that are commonly used in traditional wired sensor systems
are not tractable in WSSNs due to the limitation in wireless data communication speeds;
bringing all data to a centralized location will result in severe data congestion in the WSSN.
This issue should be resolved for the WSSN to be more widely deployed on large-scale civil
structures.

A number of decentralized approaches have been proposed for conducting SHM and
damage detection on networks of smart sensors. One of the decentralized approaches uses
independent data processing, as shown in Figure 1(b) [5–9]. Each sensor node processes
measured data independently without communicating with other sensor nodes. The processed
data, typically FFT or damage assessment results, is then sent to the base station. While the total
amount of transferred data in the network is relatively small, important spatial information (e.g.
mode shape) cannot be extracted. The decentralized approach proposed by Gao et al. [10]
employs a coordinated computing strategy as shown in Figure 1(c), which has the ability to
capture local spatial information. The sensor network in this scheme is divided into hierarchical
communities, in which sensor nodes within each community communicate with each other in
processing data; communication between communities is conducted through each community’s manager node.

While a number of decentralized algorithms for SHM and damage detection have been published, relatively little effort has been devoted to developing such approaches for modal analysis. Zimmerman et al. [11] implemented a decentralized data processing scheme on a WSSN to identify the vibration characteristics of the balcony in a historic theater in metropolitan Detroit. In the WSSN, natural frequencies were determined at each node by the peak picking method [12,13] and sent to a central node where the final natural frequencies are decided. Based on the identified natural frequencies, phase differences between the responses of each pair of sensor nodes are determined in a sequential manner (see Figure 2) and collected centrally to assemble the global mode shapes. While this decentralized algorithm was successfully implemented and tested, the approach requires a linear network Topology and may result in substantial accumulation of errors in the global mode shapes.
This paper proposes an automated hierarchical decentralized approach for modal analysis that reliably determines the global modal properties using a decentralized hierarchical network Topology. The proposed method estimates local features within each local community; global modal properties are then calculated based on the collected local information. Herein, various sensor topologies for more accurate estimation of the global mode shape are considered. Numerical simulation of a plate and truss is provided to demonstrate the effectiveness of the proposed approach.

2. PROBLEM FORMULATION

The proposed approach consists of two parts: (i) local feature extraction and (ii) determination of global modal properties. In the WSSN, local features are estimated in each local sensor community independently and subsequently collected at the base station, where the global modal properties are determined. This section describes this process.

2.1. Local feature extraction

Consider the structure and sensor network Topology depicted in Figure 3. The structure is divided into overlapping subdomains, represented by $\Omega_i (i = 1, \ldots, n)$. Data aggregation and processing are conducted independently within each subdomain. In this study, two cases regarding the input excitation are considered. In the first, the input excitation is assumed to be measurable, allowing the transfer function to be estimated. By taking the inverse FFT of the transfer function, the impulse response function can be obtained, and subsequently used as input to the Eigensystem Realization Algorithm (ERA) [14]. The second case assumes that the input excitation is unavailable, and the Natural Excitation Technique (NExT) [15] is employed. In this approach, the cross-correlation functions between the measured responses are used as the input to ERA. In both cases, only the identified local information is collected centrally for determination of the global modal properties. More information regarding implementation of these two approaches on WSSNs can be found in Nagayama and Spencer [16].

Figure 3. Structure and overlapping subdomains ($i, j = 1 \sim n$).
2.2 Determination of global modal properties

Once the local information is collected centrally, the first task is to delineate the true modes from the noise modes. In this study, the true modes are selected based on the number of identified natural frequencies from the subdomains [11]. The true modes should be identified in the majority subdomains, while the noise modes will randomly appear in the subdomains. Thus, if a specific natural frequency is identified in a substantial number of the subdomains, it is considered as a true mode. If ERA fails to find the true mode in certain subdomains, the cross spectrum is alternatively used to estimate the local mode shapes. Once the true modes are determined, the corresponding mode shapes can be combined together; the remainder of this section describes this process.

Consider global mode shape \( \phi_{\Omega}^m \) for the \( m \)th mode, along with the previously determined local mode shapes \( \phi_{\Omega_i}^m, \phi_{\Omega_2}^m, \ldots, \phi_{\Omega_n}^m \) associated with respective subdomains. The local mode shapes \( \phi_{\Omega_i}^m \) and \( \phi_{\Omega_j}^m \) associated with two neighboring subdomains can be expressed as

\[
\phi_{\Omega_i}^m = \begin{bmatrix}
1 \\
\phi_{i,2} \\
\vdots \\
\phi_{i,p} \\
\phi_{o,i}^o \\
\vdots \\
\phi_{o,i,r}^o
\end{bmatrix} \quad \text{and} \quad \phi_{\Omega_j}^m = \begin{bmatrix}
1 \\
\phi_{j,2} \\
\vdots \\
\phi_{j,q} \\
\phi_{o,j}^o \\
\vdots \\
\phi_{o,j,r}^o
\end{bmatrix}
\]

(1)

where the superscript \( o \) denotes the overlapping node in the \( i \)th and \( j \)th subdomains, \( r \) is the number of the overlapping nodes, and \( p \) and \( q \) are the number of non-overlapping nodes in the \( i \)th and \( j \)th subdomains, respectively. To allow assembly, the mode shapes in Equation (1) should be rescaled to have the same values at the overlapping nodes, i.e.

\[
R_i \begin{bmatrix}
\phi_{o,i}^o \\
\vdots \\
\phi_{o,i,r}^o
\end{bmatrix} = R_j \begin{bmatrix}
\phi_{o,j}^o \\
\vdots \\
\phi_{o,j,r}^o
\end{bmatrix}
\]

(2)

where \( R_i \) and \( R_j \) are the normalization factors for the mode shapes \( \phi_{\Omega_i}^m \) and \( \phi_{\Omega_j}^m \), respectively. The global mode shape is the union of the local mode shapes as

\[
\phi_{\Omega}^m = \bigcup_{i=1}^n R_i \phi_{\Omega_i}^m
\]

(3)

In the presence of noise, the solution to Equation (2) for any \( r > 1 \) does not exist in general. Therefore, the normalization factor \( R_i \ (i = 1, 2, \ldots, n) \) must be approximately determined, for example as a solution in the least-square sense. Because the subdomains are interconnected,
Equation (2) can be expanded up to \( n(n - 1)/2 \) equations for all pairs of the overlapping local mode shapes as follows:

\[
\phi_{\Omega_1,\Omega_2}^m = R_2 \phi_{\Omega_3,\Omega_1}^m + \varepsilon_{12} \\
\phi_{\Omega_1,\Omega_3}^m = R_2 \phi_{\Omega_2,\Omega_1}^m + \varepsilon_{13} \\
\vdots \\
\phi_{\Omega_1,\Omega_n}^m = R_2 \phi_{\Omega_n,\Omega_1}^m + \varepsilon_{1n} \\
R_2 \phi_{\Omega_2,\Omega_3}^m = R_3 \phi_{\Omega_1,\Omega_3}^m + \varepsilon_{23} \\
\vdots \\
R_2 \phi_{\Omega_2,\Omega_n}^m = R_3 \phi_{\Omega_1,\Omega_n}^m + \varepsilon_{2n} \\
\vdots \\
R_{n-1} \phi_{\Omega_{n-1},\Omega_n}^m = R_n \phi_{\Omega_n,\Omega_{n-1}}^m + \varepsilon_{(n-1)n}
\]

where \( \phi_{\Omega_1,\Omega_i}^m \) is the \( m \)th local mode shape in the \( i \)th domain at the nodes that overlap the \( j \)th domain and \( \varepsilon_{ij} \) is the error between the mode shapes. Note that the normalization factor for the first subdomain \( R_1 \) is selected to be 1. In matrix form, Equation (4) becomes

\[
y = XR + \varepsilon
\]

where

\[
y = \begin{pmatrix}
\phi_{\Omega_1,\Omega_2}^m \\
\phi_{\Omega_1,\Omega_3}^m \\
\vdots \\
\phi_{\Omega_{n-1},\Omega_n}^m \\
\end{pmatrix}, \quad X = \begin{pmatrix}
\phi_{\Omega_1,\Omega_1}^m & 0 & \cdots & 0 \\
0 & \phi_{\Omega_1,\Omega_1}^m & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \phi_{\Omega_1,\Omega_1}^m \\
-\phi_{\Omega_1,\Omega_2}^m & -\phi_{\Omega_1,\Omega_3}^m & \cdots & 0 \\
0 & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
0 & \vdots & \vdots & -\phi_{\Omega_{n-1},\Omega_n}^m \\
\end{pmatrix}, \quad R = \begin{pmatrix}
R_2 \\
R_3 \\
\vdots \\
R_{n-1} \\
\end{pmatrix}, \quad \text{and} \quad \varepsilon = \begin{pmatrix}
\varepsilon_{12} \\
\varepsilon_{13} \\
\vdots \\
\varepsilon_{2n} \\
\varepsilon_{(n-1)n}
\end{pmatrix}
\]

The estimator \( \hat{R} \) that minimizes the square of errors \( \varepsilon^T \varepsilon \) is given by [17]

\[
\hat{R} = (X^TX)^{-1}X^Ty
\]
Using the normalization factor $R$, the local mode shapes are scaled and assembled to obtain the global mode shape. At the overlapping nodes, the local mode shapes are averaged to obtain the associated values of the global mode shape.

![Figure 4. 4 DOF model and mode shape: (a) 4 DOF model and (b) exact global mode shape (first mode).](image)

![Figure 5. Topologies with different numbers of overlapping nodes: (a) Topology 1: one overlapping node and (b) Topology 2: two overlapping nodes.](image)

<table>
<thead>
<tr>
<th>Table I. Global and local mode shapes.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Mode shape</td>
</tr>
<tr>
<td>Exact global mode shape</td>
</tr>
<tr>
<td><em>Local mode shape w/10% error</em></td>
</tr>
<tr>
<td>Topology 1</td>
</tr>
<tr>
<td>Group 1</td>
</tr>
<tr>
<td>Group 2</td>
</tr>
<tr>
<td>Group 3</td>
</tr>
<tr>
<td>Group 4</td>
</tr>
<tr>
<td>Topology 2</td>
</tr>
<tr>
<td>Group 1</td>
</tr>
<tr>
<td>Group 2</td>
</tr>
<tr>
<td>Group 3</td>
</tr>
</tbody>
</table>

The accuracy of the combined global mode shapes can be evaluated by the error between the combined and reference global mode shapes, $\phi^m_\Omega$ and $\phi^m_{\Omega,\text{ref}}$, respectively:

$$e_m = \phi^m_\Omega - \phi^m_{\Omega,\text{ref}}$$

(7)

The reference global mode shapes are estimated using all sensor data simultaneously. Note that the reference global mode shapes correspond to the centralized data acquisition and processing scheme commonly used in the wired sensor networks, while the proposed decentralized approach is for the WSSN. For the error $e_m$ to be meaningful, the two global mode shapes should be appropriately normalized. Thus, $\phi^m_{\Omega,\text{ref}}$ is normalized so that the largest element in $\phi^m_{\Omega,\text{ref}}$ is equal to 1, and $\phi^m_\Omega$ is then scaled to minimize the square error between $\phi^m_\Omega$ and $\phi^m_{\Omega,\text{ref}}$:

$$\min(\phi^m_\Omega - \phi^m_{\Omega,\text{ref}})^T(\phi^m_\Omega - \phi^m_{\Omega,\text{ref}})$$

(8)

To better understand the proposed method that employs the least-squares approximation, consider the 5 DOF spring-mass model in Figure 4(a) and its first mode shape in Figure 4(b). This model is divided into subgroups forming two different topologies as shown in Figure 5. Note that subgroups share one overlapping node in Topology 1 and two in Topology 2. Table I summarizes the global and local mode shapes normalized with respect to the first node in each

Figure 6. Comparison of the global mode shapes: (a) assembled mode shapes and (b) errors of assembled mode shapes.

mode shape and with 10% error. In Topology 2, the normalization factors calculated using Equation (6) are $R_2 = 2.0694$ and $R_3 = 2.7388$. The scaled local mode shapes are

$$\phi^1_{\Omega_i} = \begin{pmatrix} 1.0000 \\ 2.2000 \\ 2.7500 \end{pmatrix}, \quad R_2\phi^1_{\Omega_2} = \begin{pmatrix} 2.0694 \\ 2.8454 \\ 2.2763 \end{pmatrix}, \quad R_3\phi^1_{\Omega_3} = \begin{pmatrix} 2.7388 \\ 2.4102 \\ 1.2051 \end{pmatrix} \quad (9)$$

Taking the average values at the overlapping nodes, the global mode shape is assembled as

$$\phi^1_{\Omega} = (1.0000 \ 2.1347 \ 2.7781 \ 2.3433 \ 1.2051)^T \quad (10)$$

where $T$ denotes the matrix transpose. The same procedure can be applied to Topology 1, resulting in the global mode shape as follows:

$$\phi^1_{\Omega} = (1.0000 \ 2.2000 \ 3.0250 \ 2.6620 \ 1.4641)^T \quad (11)$$

Note that the averaging is not required to assemble the scaled local mode shapes in Topology 1 because the subgroups share only one node.

Figure 6 compares the exact global mode shape and the combined global mode shapes of topologies 1 and 2. After normalizing the combined and reference global mode shapes, which is
the exact in this example, the error $e_1$ is calculated. From Figure 6, the combined global mode shape of Topology 2 is seen more accurate than that of Topology 1. In the subsequent sections, numerical examples are provided to investigate the efficacy of the proposed method in detail, mainly focused on the sensor topologies.

3. NUMERICAL EXAMPLES

Numerical examples of a plate and planar truss are provided to validate the proposed approach. In these examples, the effects of the sensor topologies on the accuracy of the combined global mode shapes are investigated.

3.1. Plate model

3.1.1. Numerical model. Consider the uniform plate shown in Figure 7. The top and left edges of the plate are considered as fixed, and the bottom and right edges are simply supported. The plate

![Global mode shapes from the finite element model.](image)

$\text{Figure 8. Global mode shapes from the finite element model.}$

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is assumed to be 0.7 m by 1 m and made of steel having an elastic modulus of 200 GPa, a Poisson ratio of 0.3, mass density of $7.83 \times 10^3$ kg/m$^3$, and thickness of 1 mm. The numerical model of the plate is implemented in MATLAB using the 12 DOF (3 DOF at each node) rectangular Kirchhoff plate element known as the ACM element [18]. The shape functions are selected to be incomplete, 4th-order polynomials in the $x$- and $y$-directions with 12 terms. The first seven out-of-plane mode shapes of the plate model are presented in Figure 8, and the transfer functions between the input excitation applied vertically at the nodes marked by an ‘X’ and the accelerations at nodes N1 and N2 (see Figure 7) are shown in Figure 9.

<table>
<thead>
<tr>
<th>Excitation type</th>
<th>Input measured</th>
<th>Input not measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impulse</td>
<td>Case 1 (ERA)</td>
<td>Case 2 (ERA)</td>
</tr>
<tr>
<td>Band-limited white noise</td>
<td>Case 3 (ERA)</td>
<td>Case 4 (NExT/ERA)</td>
</tr>
</tbody>
</table>

Figure 9. Transfer functions (circles represent natural frequencies): (a) N1 and (b) N2.

Figure 10. Schematic view of subdomains (dashed line): (a) Topology 1: two nodes in each group; (b) Topology 2: four nodes in each group; and (c) Topology 3: nine nodes in each group.
A total of 63 evenly spaced sensor nodes are selected as shown in Figure 7 to obtain vertical accelerations under the input excitation. As summarized in Table II, four different simulation cases are considered: an impulse loading (cases 1 and 2) or a band-limited white noise up to 50 Hz (cases 3 and 4). Five percent RMS noise is added to all measurements. ERA is used to identify the mode shapes and frequencies in cases 1, 2, and 3, whereas NExT/ERA is used in case 4. As the input excitation to large-scale civil engineering structures is not generally available, output-only modal analysis for random excitations is often employed; thus, case 4 may be considered as the most important for SHM.

3.1.2. Sensor Topology. To investigate the effect of the sensor topologies on the combined global mode shapes, the nodes are grouped in four different ways, as shown in Figure 10: (a) Topology 1: two nodes in each group with one overlapping node, (b) Topology 2: four nodes in each group with two overlapping nodes, and (c) Topology 3: nine nodes in each group with three overlapping nodes. Note that Topology 1 corresponds to the approach adopted by Zimmerman et al. [11].

From the data communication perspective, having fewer local groups is advantageous due to less amount of data being transmitted over the radio. In a local group, the reference sensor sends sensor data that are generally long and all other sensors transmit the condensed data such as the correlation function of the cross spectrum. Thus, the amount of transmitted data can be drastically reduced by adopting a Topology with less number of local groups. Assuming the time history record of length \( N \) and \( n_d \) times averaging without any overlap between spectral windows, the total transmitted data [16] are at most

\[
N_{\text{data}} = n_g \left( N \cdot n_d + \frac{N}{2} \cdot (n_s - 1) \right)
\]

where \( n_g \) and \( n_s \) are the numbers of local groups and sensor nodes, respectively. From Equation (12) with \( N = 1024 \), \( N_{\text{data}} \) are about \( 1.30 \times 10^6 \), \( 1.06 \times 10^6 \), and \( 0.29 \times 10^6 \) for Topology 1, 2, and 3, respectively. Thus, Topology 3 has the least data communication requirement.

3.1.3. Determination of true modes. As previously described, the number of groups in which a natural frequency is locally identified is utilized to delineate between true modes and noise modes. The natural frequencies estimated in each group are collected, and if the number of collected natural frequencies in a specific frequency range is greater than a predetermined

Figure 11. Number of identified natural frequencies (case 4, random excitation and input not measured): (a) Topology 1; (b) Topology 2; and (c) Topology 3.
threshold value, it is assumed to be a true mode. The threshold is selected to be 70% of the number of local groups. The frequency range in which identified frequencies are counted is \((f_c - \Delta f, f_c + \Delta f)\) where \(f_c\) is the central frequency of the range and \(2\Delta f\) is the width. In this study, \(f_c\) and \(\Delta f\) are selected as follows:

\[
f_c = k \cdot \frac{f_s}{N_{\text{FFT}}} \quad k = 1, 2, \ldots
\]

\[
\Delta f = 2 \cdot \frac{f_s}{N_{\text{FFT}}}
\]

where \(f_s\) is the sampling rate, \(N_{\text{FFT}}\) is the number of FFT points, and \(k\) is any positive integer such that \(f_c\) is less than the bandwidth of the input excitation. Note that the adjacent ranges are made to overlap with each other to prevent the case that the identified frequencies are evenly distributed over two adjacent non-overlapping ranges.

Figure 11 shows the typical number of groups in which a natural frequency is found in case 4 (random excitation, input not measured). The identified frequencies in the local groups are concentrated at several frequencies such as about 10, 18, 27, 31, 34, 47, and 49 Hz that are finally considered as corresponding to true modes. In some frequency ranges, the number of local groups is greater than the total number of local groups due to noise modes that are closely located to the true modes. In this case, the frequency that is closest to \(f_c\) is considered as the natural frequency of the local group.

### 3.1.4. Global mode shapes.

The global mode shapes can be assembled with the local mode shapes using the proposed method described in Section 2.2. If a true mode is not identified in a certain local group, cross spectra of the accelerations in the group are alternatively utilized to estimate the local mode shapes. The cross spectrum values at the spectral line nearest to the natural frequency of the majority groups are assumed as the mode shape. Note that mode shapes obtained only from the cross spectra may not be as accurate as those from the ERA because the cross spectra have values only at the spectral lines.

The error \(e_j\) defined in Equation (7) is calculated to assess the accuracy of the combined global mode shape. Typical plots of the error \(e_2\) for each Topology in case 4 are shown in Figure 12. Improvement in accuracy is clearly shown for Topologies 2 and 3, where larger local groups and more overlapping nodes are employed.

![Figure 12. Absolute value of the errors between the combined and reference global mode shapes (case 4, second mode): (a) Topology 1; (b) Topology 2; and (c) Topology 3.](image-url)
Two error measures are considered to quantitatively evaluate the accuracy of the combined global mode shapes. The first is the maximum error for the \( j \)th mode:

\[
E_{\text{max},j} = \max(|e_{j,1}|, |e_{j,2}|, \ldots, |e_{j,n}|)
\]  

(14)

where \( n \) is the number of elements in \( e_j \), and \( e_{j,i} \) are the \( i \)th element of the error \( e_j \). The second error measure is the average error for the \( j \)th mode:

\[
E_{\text{avg},j} = \text{mean}(|e_{j,1}|, |e_{j,2}|, \ldots, |e_{j,n}|)
\]  

(15)

If a mode is not identified, the error of the mode is set to 100%.

Repeating the simulation 100 times for each case, the averages of each error measure are calculated for the first seven modes as shown in Figures 13 and 14. Zoomed figures from 0 to 15% are also provided in Figure 14 to clearly show \( E_{\text{avg},j} \). In Figures 13 and 14, Topology 3 consistently has the smallest errors in most cases. It can be concluded that sufficiently large local groups and multiple overlapping nodes as in Topology 3 contribute to reliable and accurate estimation of global modal properties.
3.2. Truss model

3.2.1. Numerical model. Consider the three-dimensional truss model shown in Figure 15. This simply supported truss consists of 53 elements that have the identical sectional and material properties shown in Table III. The input excitation, either impulse loading or random

Figure 14. Average error: (a) case 1: impulse loading, input measured; (b) case 2: impulse loading, input not measured; (c) case 3: random excitation, input measured; and (d) case 4: random excitation, input not measured.
Figure 15. Truss model.

Table III. Sectional and material properties.

<table>
<thead>
<tr>
<th>Cross-sectional area (m²)</th>
<th>$I_x$ (m⁴)</th>
<th>$I_y$ (m⁴)</th>
<th>$I_z$ (m⁴)</th>
<th>Elastic modulus (Pa)</th>
<th>Shear modulus (Pa)</th>
<th>Mass density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0 × 10⁻⁴</td>
<td>5.0 × 10⁻⁹</td>
<td>2.5 × 10⁻⁹</td>
<td>2.5 × 10⁻⁹</td>
<td>1.999 × 10¹¹</td>
<td>7.692 × 10¹¹</td>
<td>7.827 × 10⁵</td>
</tr>
</tbody>
</table>

Figure 16. Transfer functions (circles represent natural frequencies): (a) Node N1 and (b) Node N2.
Figure 17. Plan view of the truss model for sensor topologies: (a) Topology 1–1 overlapping node and (b) Topology 2–2 overlapping nodes.

Figure 18. Maximum error: (a) case 1: impulse loading, input measured; (b) case 2: impulse loading, input not measured; (c) case 3: random excitation, input measured; and (d) case 4: random excitation, input not measured.
excitation, is applied vertically as shown in Figure 15. Transfer functions between the excitation and the accelerations at nodes N1 and N2 marked by circles in Figure 15 are shown in Figure 16. As in the plate example, four cases in Table II are considered. For cases 3 and 4, a band-limited white noise of which bandwidth is from 0 to 120 Hz that encompasses the first four

Figure 19. Average error: (a) case 1: impulse loading, input measured; (b) case 2: impulse loading, input not measured; (c) case 3: random excitation, input measured; and (d) case 4: random excitation, input not measured.
modes of the truss is applied. Vertical accelerations at all lower nodes are obtained in each case. Five percent RMS noise is added to all measurements.

3.2.2. Sensor Topology. Two types of sensor topologies in Figure 17 are considered: (a) for Topology 1 each local group consist of two nodes, one of which is the overlapping node and (b) for Topology 2 each local group has four nodes, two of which are the overlapping nodes.

3.2.3. Determination of global modal properties. The proposed method is applied to determine the global modal properties of the truss. Modal analysis is conducted in each group using ERA (cases 1, 2, and 3) or NExT/ERA (case 4) to estimate local modal properties, and the true modes are obtained based on the locally identified natural frequencies. However, as can be seen from Figure 16, the presence of the first mode in the response is quite small, which results in it not

Figure 20. Comparison of mode shapes: third mode in case 4. (a) Topology 1 and (b) Topology 2.
being identified as a true mode for case 4. This mode can be manually identified if the correlation function is collected to the base station.

As in the plate example, the maximum and average errors between the reference and combined global mode shapes are selected to evaluate the accuracy. The simulation is repeated 100 times, and the averages of the errors are obtained for the first seven modes as shown in Figures 18 and 19. In these graphs, Topology 2 (larger local groups and multiple overlapping nodes) exhibits consistently smaller error than that of Topology 1. Particularly in case 4, which is considered as the most important for the civil structures due to the difficulty in measuring the input excitation, the difference between the errors for Topologies 1 and 2 is clearly seen in Figure 18(d) and Figure 19(d).

To visualize the difference, typically combined global mode shapes of the third, sixth, and seventh modes for Topologies 1 and 2 are compared with the reference global mode shapes in Figures 20–22. Here, the mode shape at the upper nodes is assumed as having the same values with the corresponding lower nodes to fully draw mode shapes of the truss. The discrepancy between the reference and the combined global mode shape of Topology 1 is clear; Topology 2 shows better agreement with the reference. The
proposed method is seen to combine the global mode shape reliably and accurately if sensor Topologies are appropriately selected to have sufficiently large local groups and multiple overlapping nodes.

4. CONCLUSION

An automated, hierarchical, decentralized approach for modal analysis using smart sensors has been proposed using a decentralized network Topology. The proposed approach consists of two main parts: (1) local feature extraction and (2) determination of global modal properties. Modal analysis is conducted independently in each sensor group to estimate the local modal information using ERA or NExT/ERA. Global modal properties are then obtained using the aggregated local properties in the base station. Frequency for which the local groups identify the natural frequencies is utilized to delineate between the true and noise modes. Then, local mode shapes are assembled using a least-squares approximation to estimate the global mode shape.

The proposed approach was numerically validated. From the plate and truss examples, sensor topologies were investigated in term of the size of local groups and the number of

Figure 22. Comparison of mode shapes: seventh mode in case 4. (a) Topology 1 and (b) Topology 2.
overlapping nodes. Larger groups with overlapping nodes were found to reduce errors in the assembled global mode shape. The numerical results show that the proposed approach for decentralized modal analysis is efficient for implementation on WSSNs. While ERA and NExT/ERA are used in this study to characterize the local modal properties, other system identification methods such as Stochastic Subspace Identification [19,20] or Frequency Domain Decomposition [21] with Random Decrement Technique [22,23] can be alternatively used. Implementing the proposed approach on the WSSN is currently underway.

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